Knuth-Morris-Pratt Algorithm

CS181 Fall 2020

Professor Sorin Istrail

Overview of Knuth-Morris-Pratt (KMP)

- The Knuth-Morris-Pratt (KMP) algorithm is a pattern-matching algorithm; it finds all occurrences of a pattern P of length p in a text T of length t
- It takes advantage of the failure function f on the pattern P to search in linear time $O(p + t)$!
  - The general idea is that after we've seen a character in T once, we should already be able to tell whether the pattern could start there, even if we never explicitly attempted to match P_1 directly to T_j
- We've already seen the algorithm and pseudocode for constructing the failure function, so we'll focus on KMP here using a similar example

Definitions

- Inputs:
  - Text T, indexed by j from 1 to t
  - Pattern P, indexed by i from 1 to p
- Output:
  - A list of positions k, where $T_{k\,:\,k+p} = P$
- Failure function, f
  - A table of p entries, where each entry $f(i)$ is the length of the longest proper suffix of $P_{1\,:\,i}$ which is also a proper prefix of P
  - See previous slide deck for a more detailed explanation

The Algorithm

1. Calculate the failure function *f* for the pattern *P*
2. Construct a skeleton DFA which accepts *P* and includes transitions based on *f*
3. Initialize the skeleton DFA to state 0 and the *T* pointer to 1
4. Iterate through the text *T*

**Here we show a version of the pseudocode which conceptualizes KMP with an accepting skeleton DFA. In practice, the skeleton DFA behavior can also be achieved using only the pattern *P*, the failure function *f*, and a pointer *i* which indexes symbols in *P* rather than states in *M*.

```
calculate f(i) for 1 ≤ i ≤ p
construct a skeleton DFA M for P using f
M starts in state M_0
i := current state in M (updated with transitions)
j ← 1
while j ≤ t do
    if T_j = P_{i+1} then
        j ← j + 1
        M enters state M_{i+1}
        if M is in state M_p then
            record (j − p)
            M enters state M_{f(p)}
        end
    else
        M enters state M_{f(i)}
        if M is in state M_0 and T_j ≠ P_{i+1} then
            j ← j + 1
        end
    end
end
```

An Example

T = aabbabaabaabca

P = abaabc

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
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$j \leftarrow 1$
while $j \leq t$ **do**
    if $T_j = P_{i+1}$ **then**
        $j \leftarrow j + 1$
        M enters state M_{i+1}
        if M is in state M_p **then**
            record $(j - p)$
            M enters state $M_{f(p)}$
        end
    else
        M enters state $M_{f(i)}$
        if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
            $j \leftarrow j + 1$
        end
    end
end

aabbabaabaabca

abaabc

i	1	2	3	4	5	6
P_i	a	b	a	a	b	c
$f(i)$	0	0	1	1	2	0

**See previous set of slides for exactly how we constructed this!

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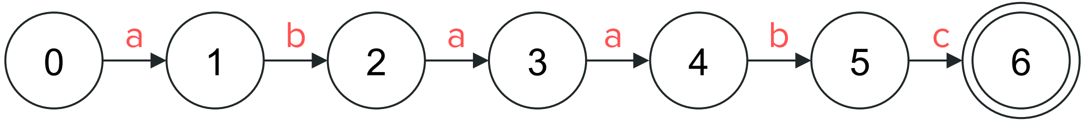

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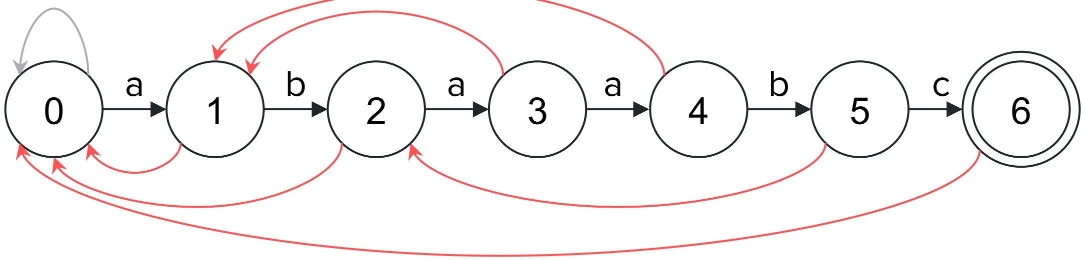

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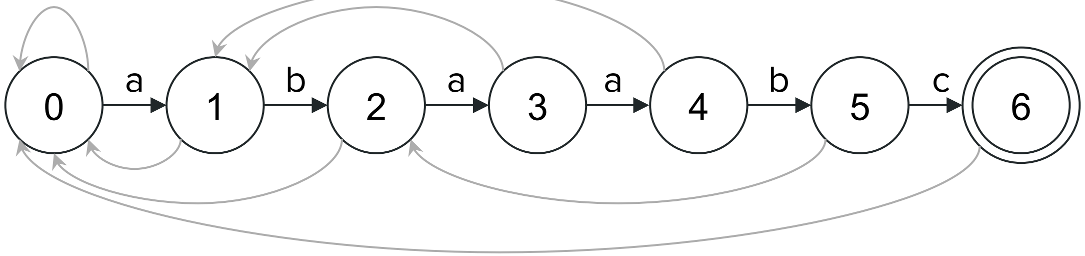

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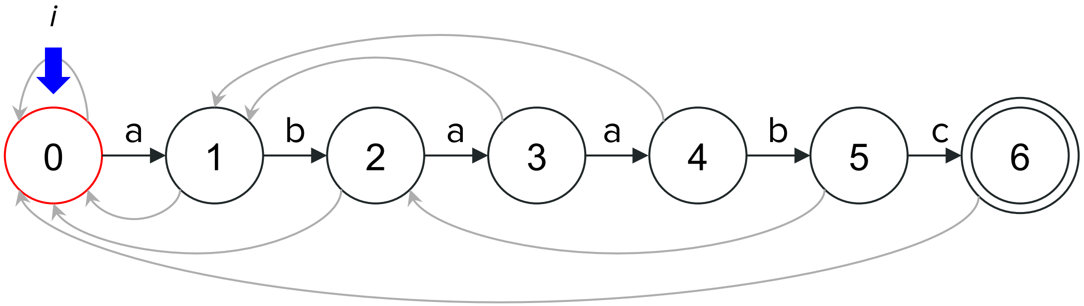

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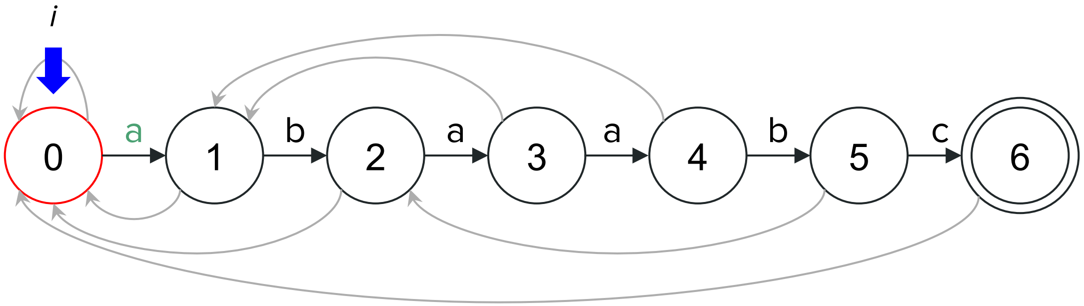

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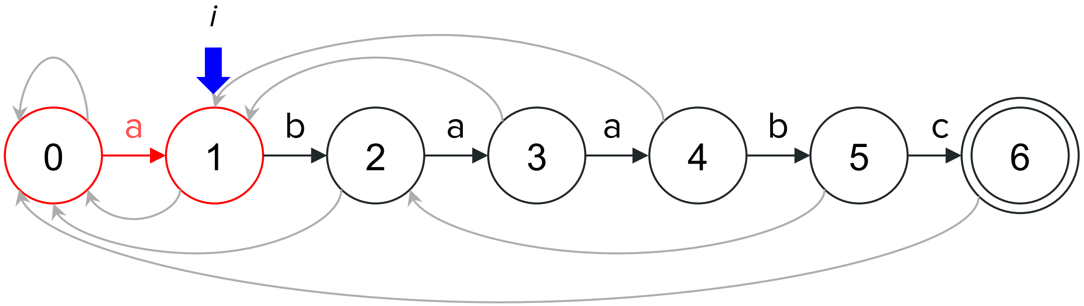

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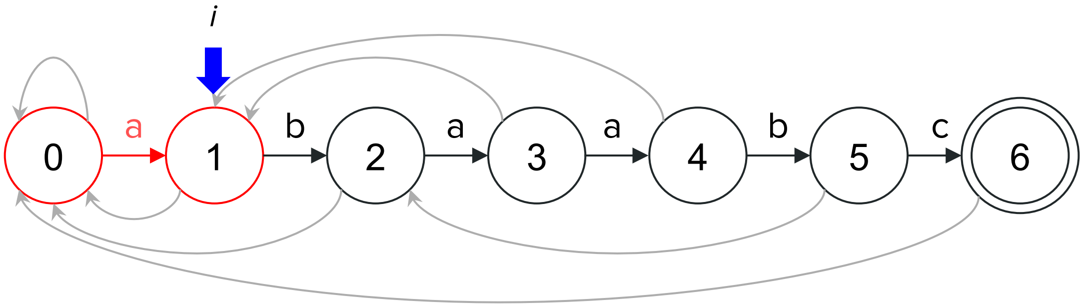

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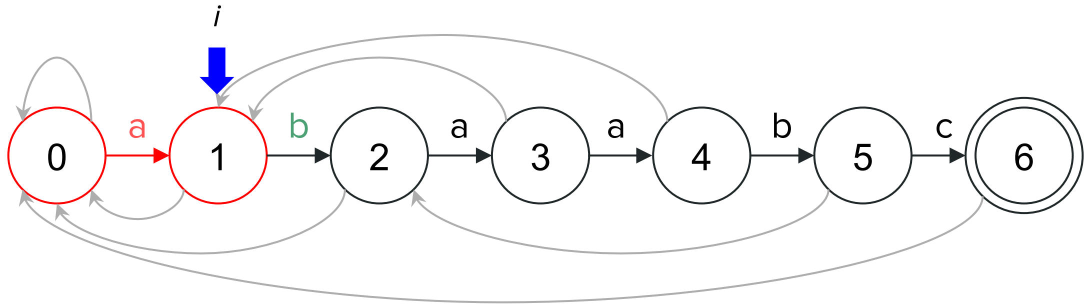

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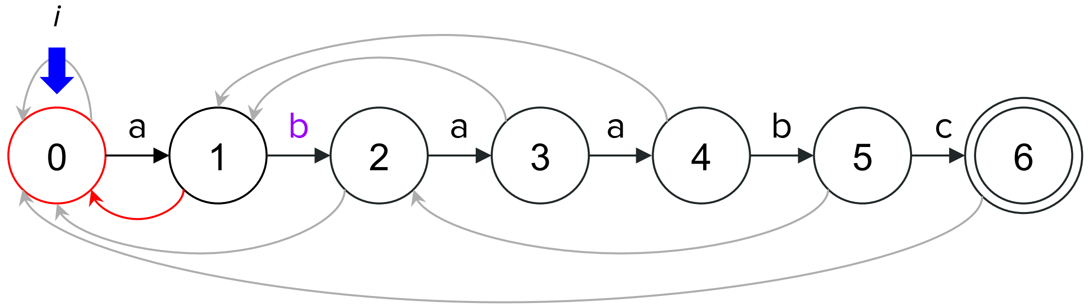

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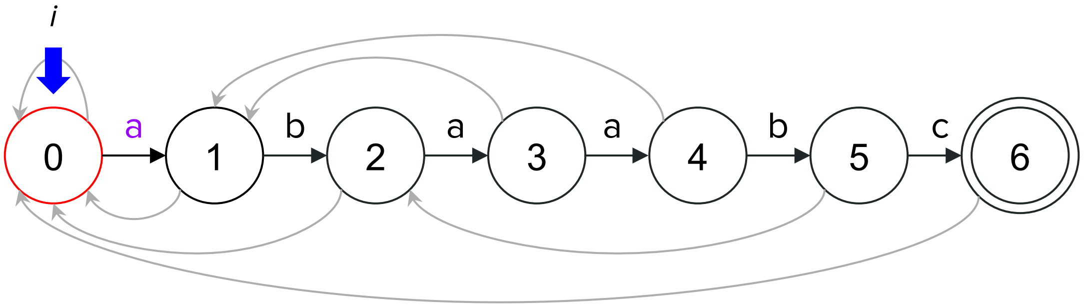

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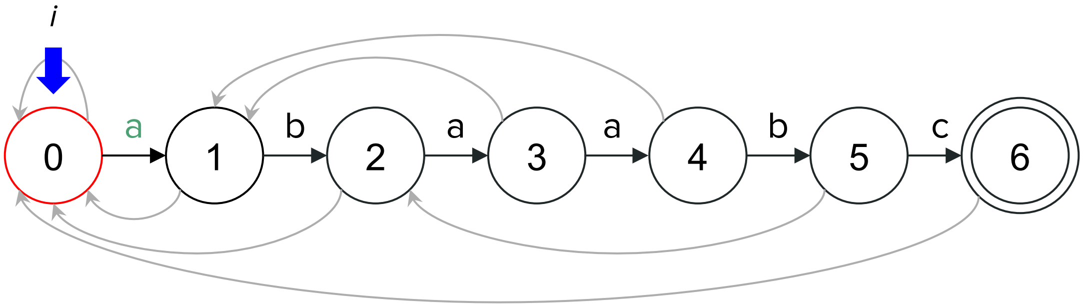

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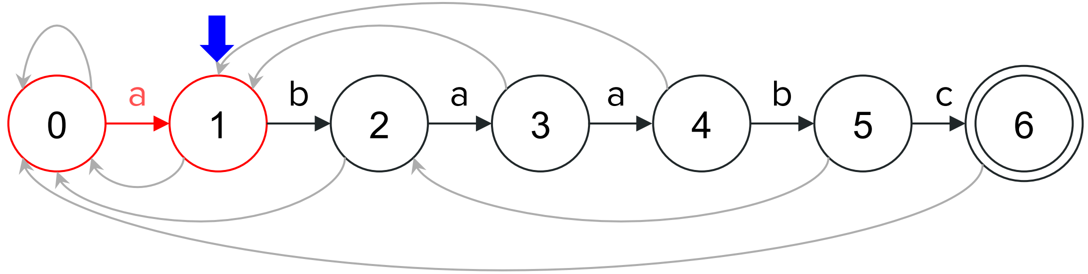

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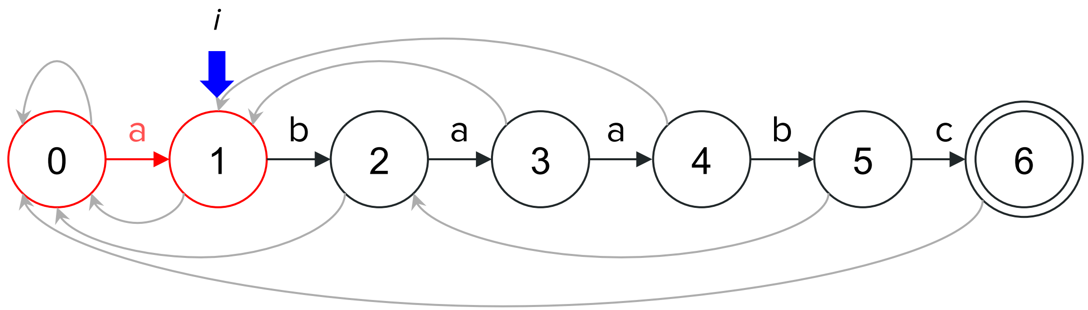

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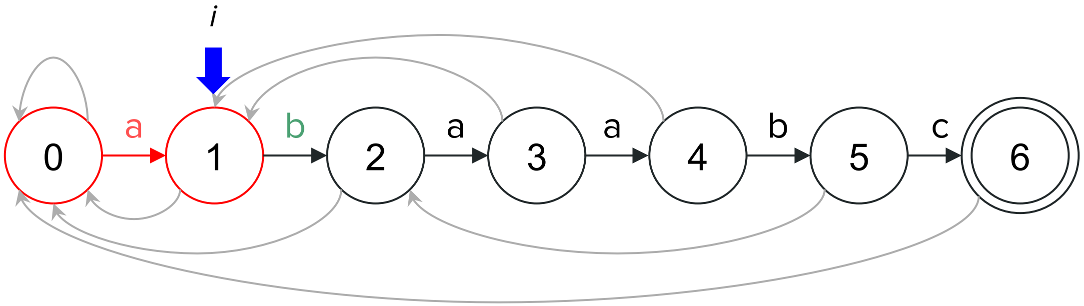

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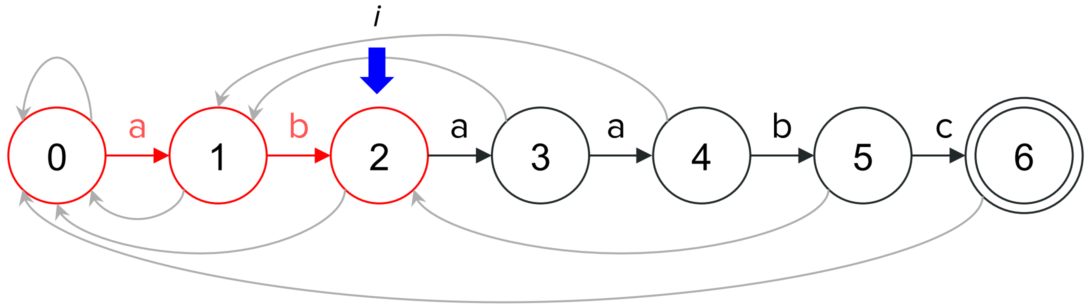

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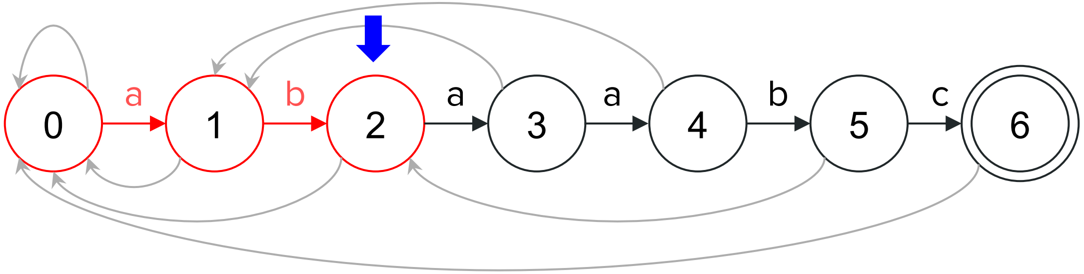

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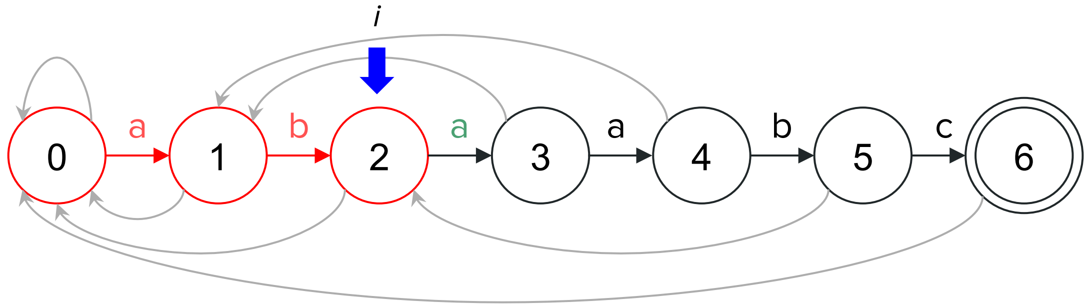

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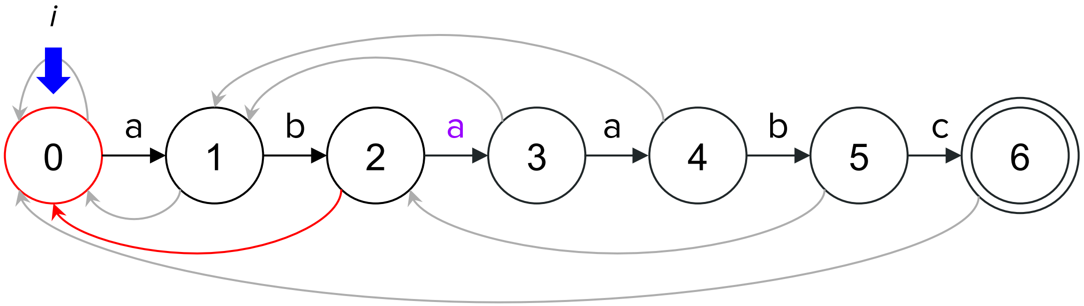

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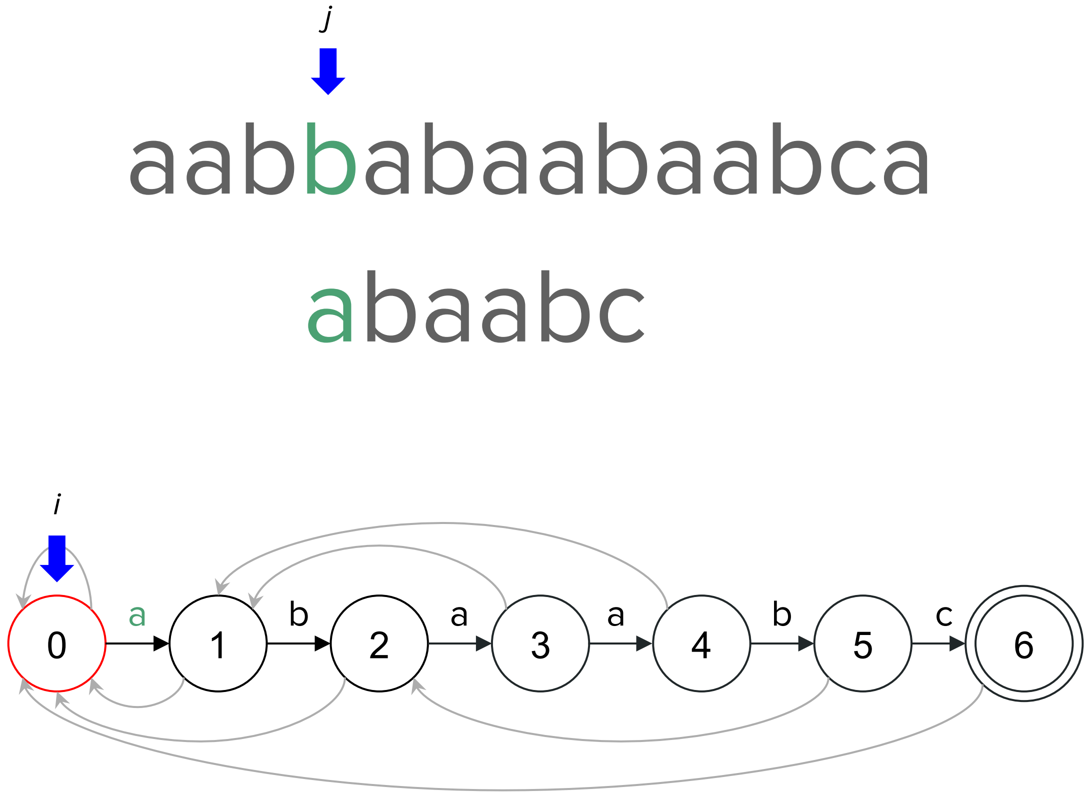

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  end
end

j

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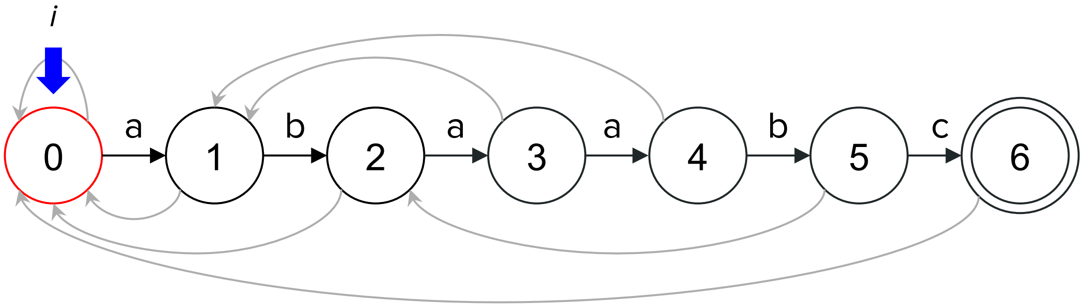

f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
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$j \leftarrow 1$
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        $j \leftarrow j + 1$
        M enters state M_{i+1}
        if M is in state M_p **then**
            record $(j - p)$
            M enters state $M_{f(p)}$
        end
    else
        M enters state $M_{f(i)}$
        if M is in state M_0 **and** $\boxed{T_j \neq P_{i+1}}$ **then**
            $\boxed{j \leftarrow j + 1}$
        end
    end
end

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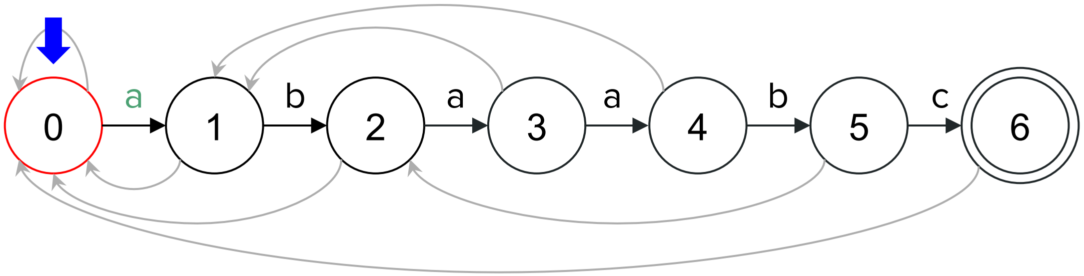

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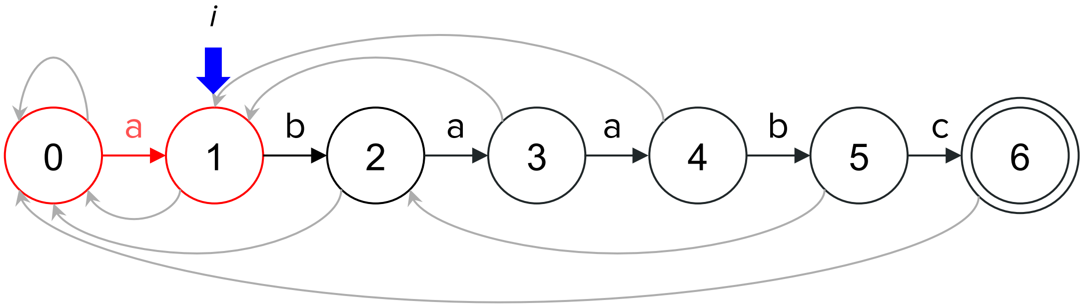

f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$

construct a skeleton DFA M for P using f

M starts in state M_0

$i :=$ current state in M (updated with transitions)

$j \leftarrow 1$

while $j \leq t$ **do**

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    $j \leftarrow j + 1$

    M enters state M_{i+1}

    if M is in state M_p **then**

      record $(j - p)$

      M enters state $M_{f(p)}$

    end

  else

    M enters state $M_{f(i)}$

    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**

      $j \leftarrow j + 1$

    end

  end

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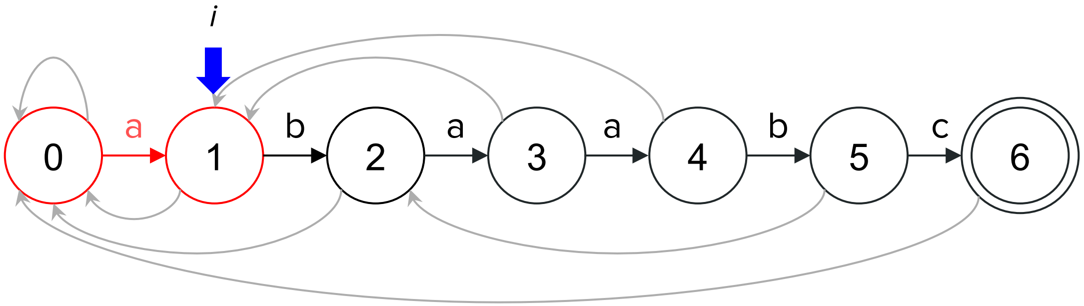

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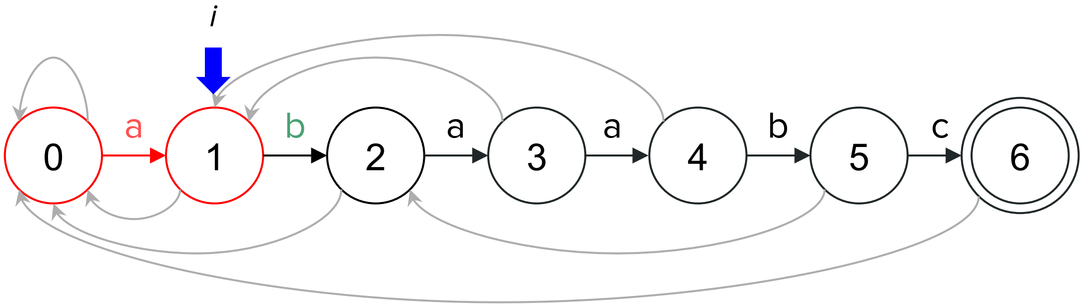

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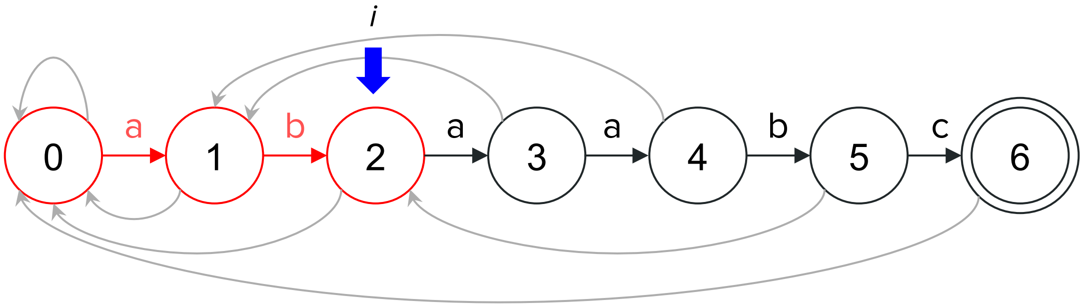

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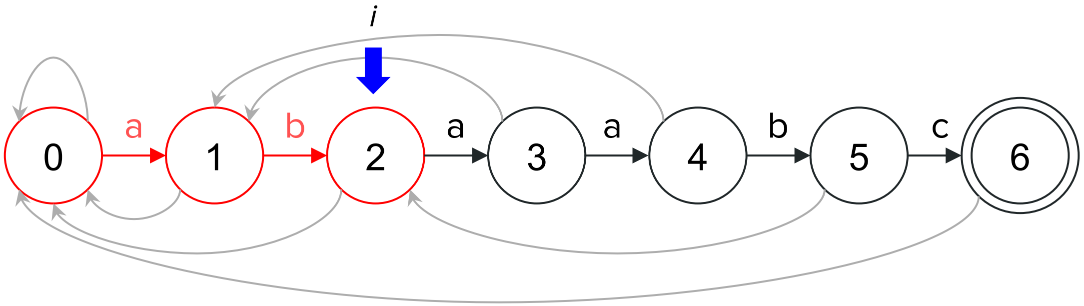

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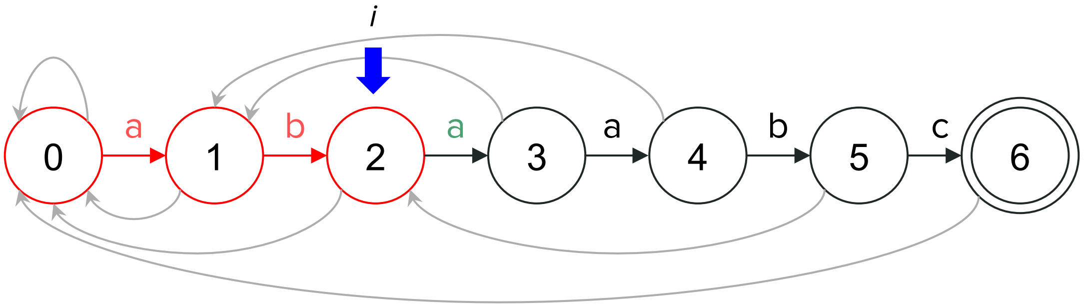

f(i)	0	0	1	1	2	0

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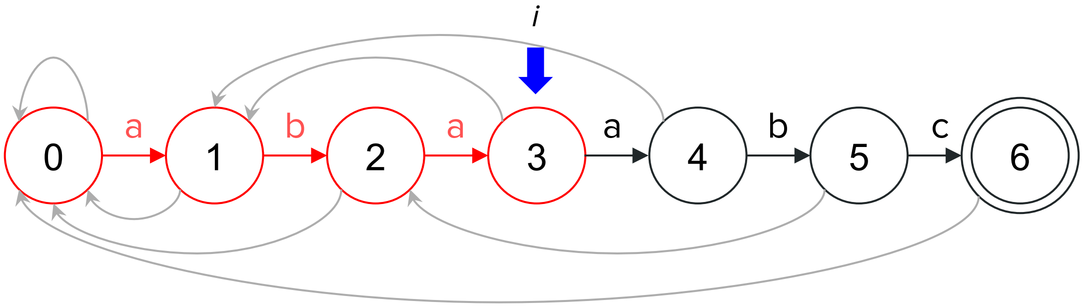

f(i)	0	0	1	1	2	0

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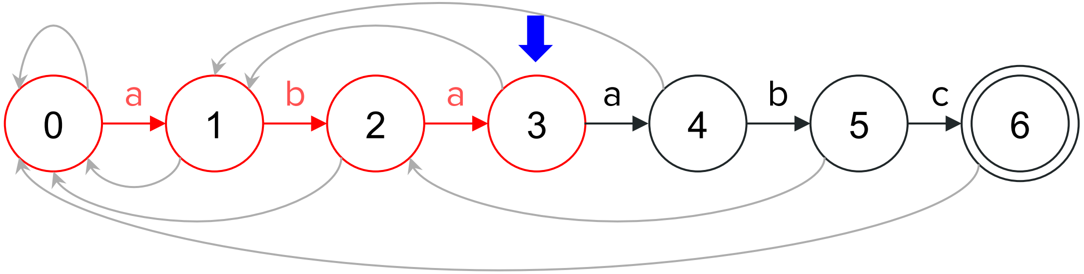

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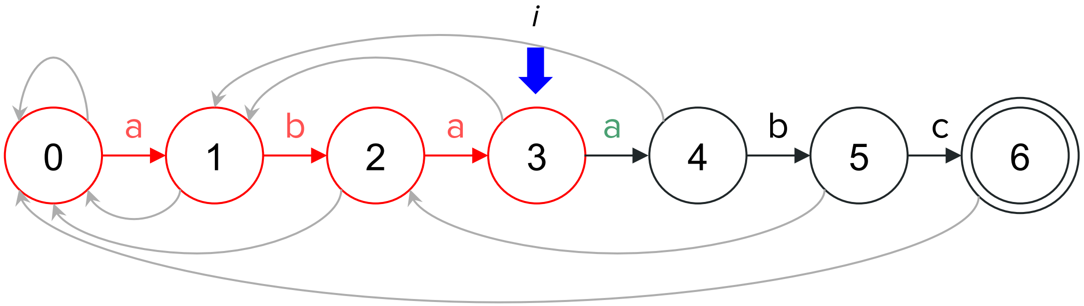

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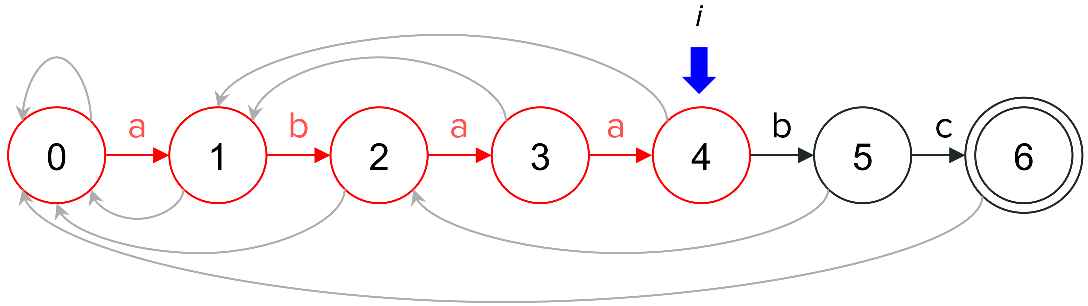

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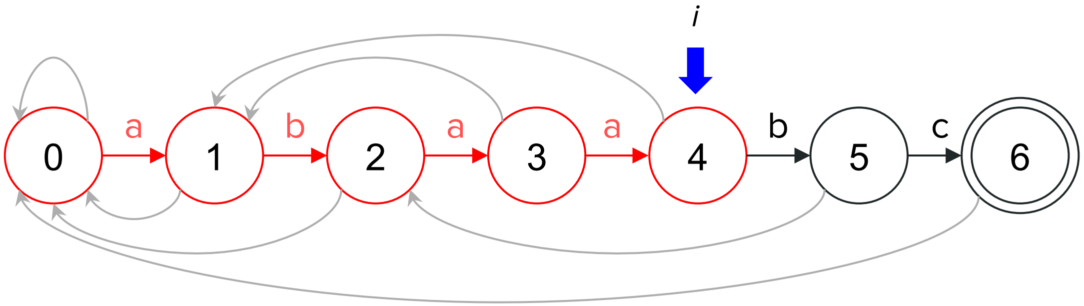

f(i)	0	0	1	1	2	0

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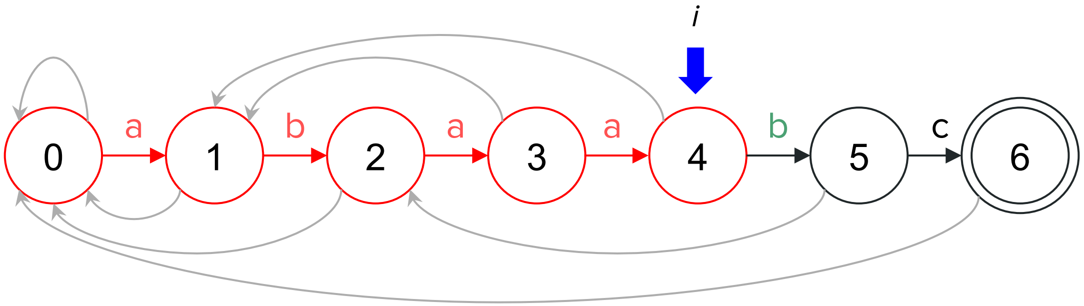

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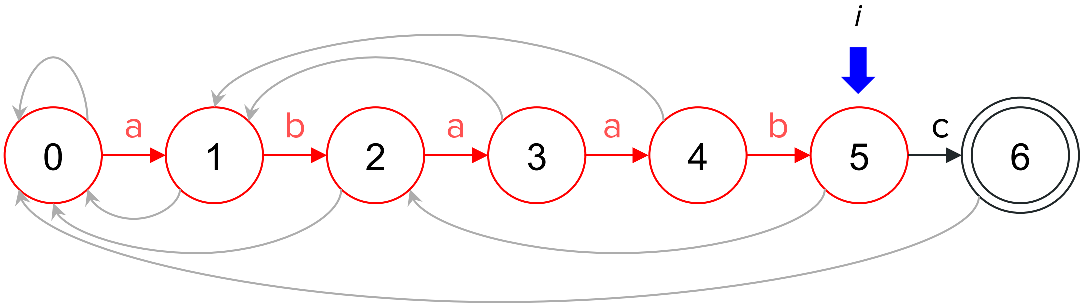

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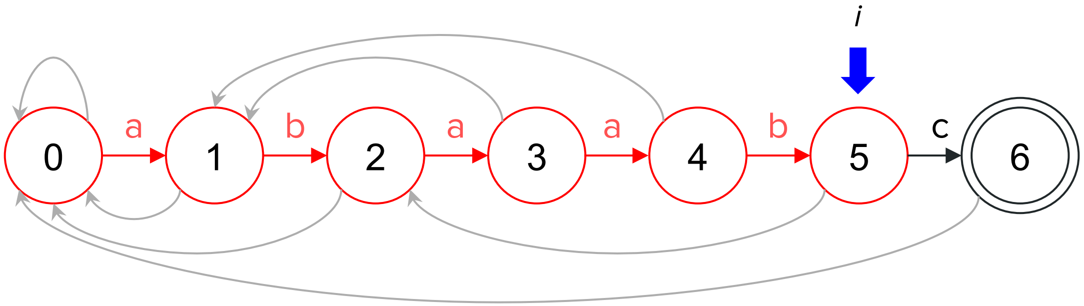

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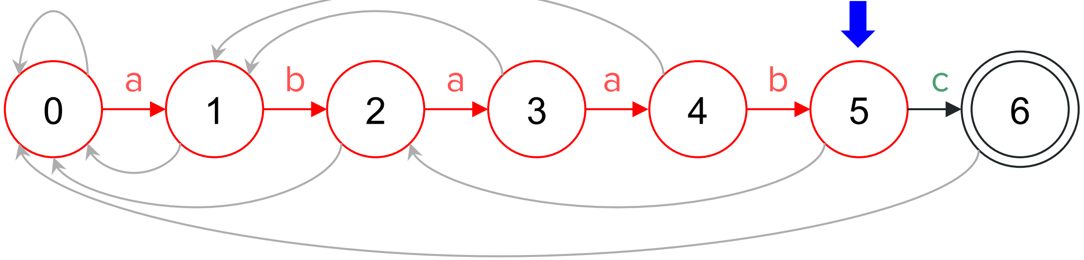

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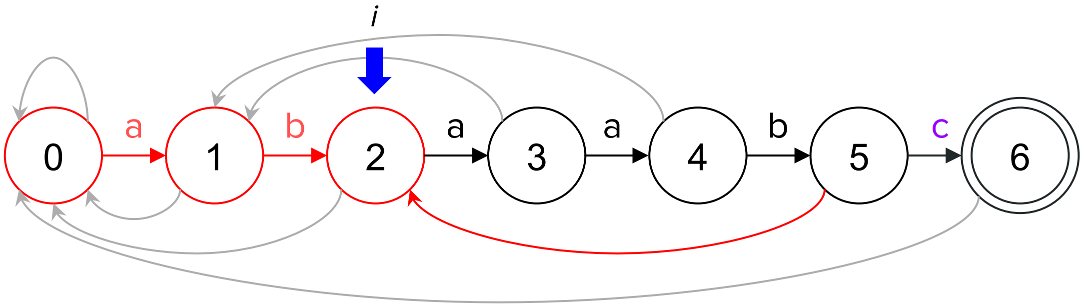

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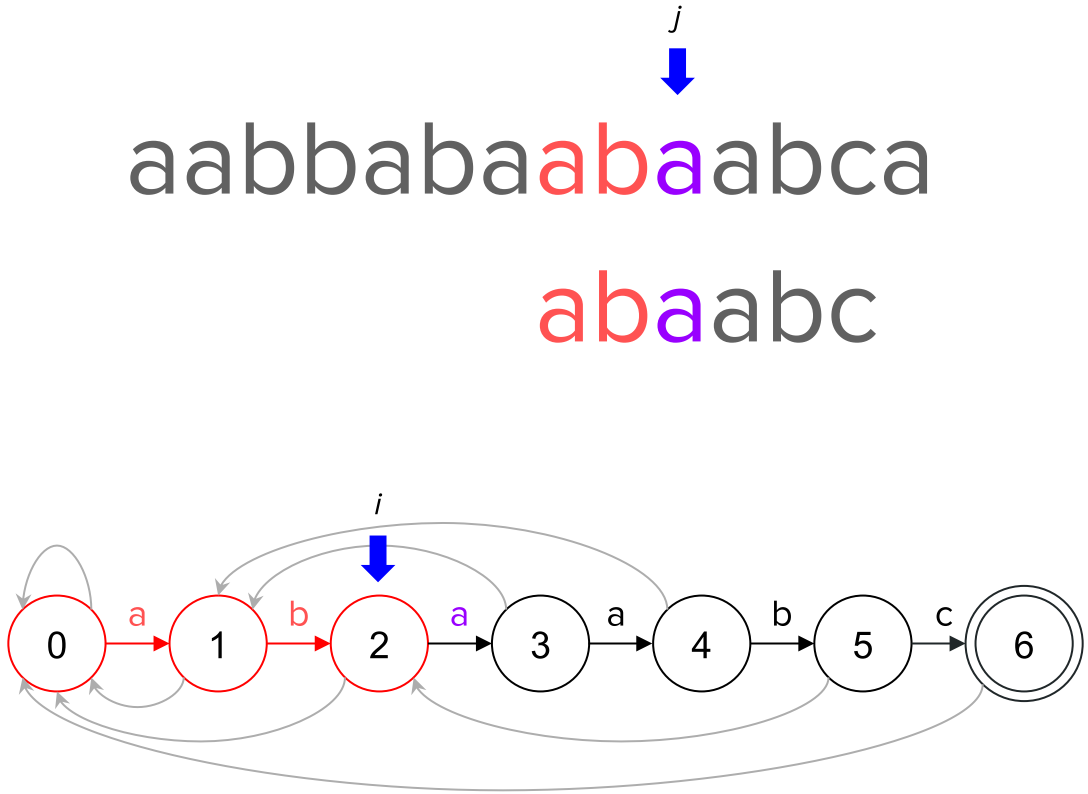

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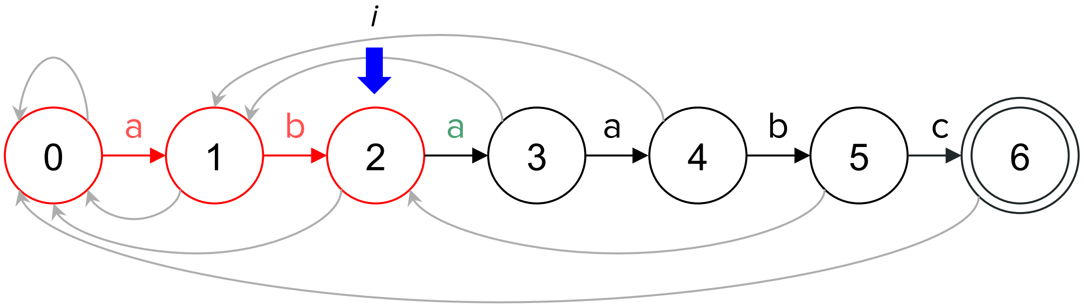

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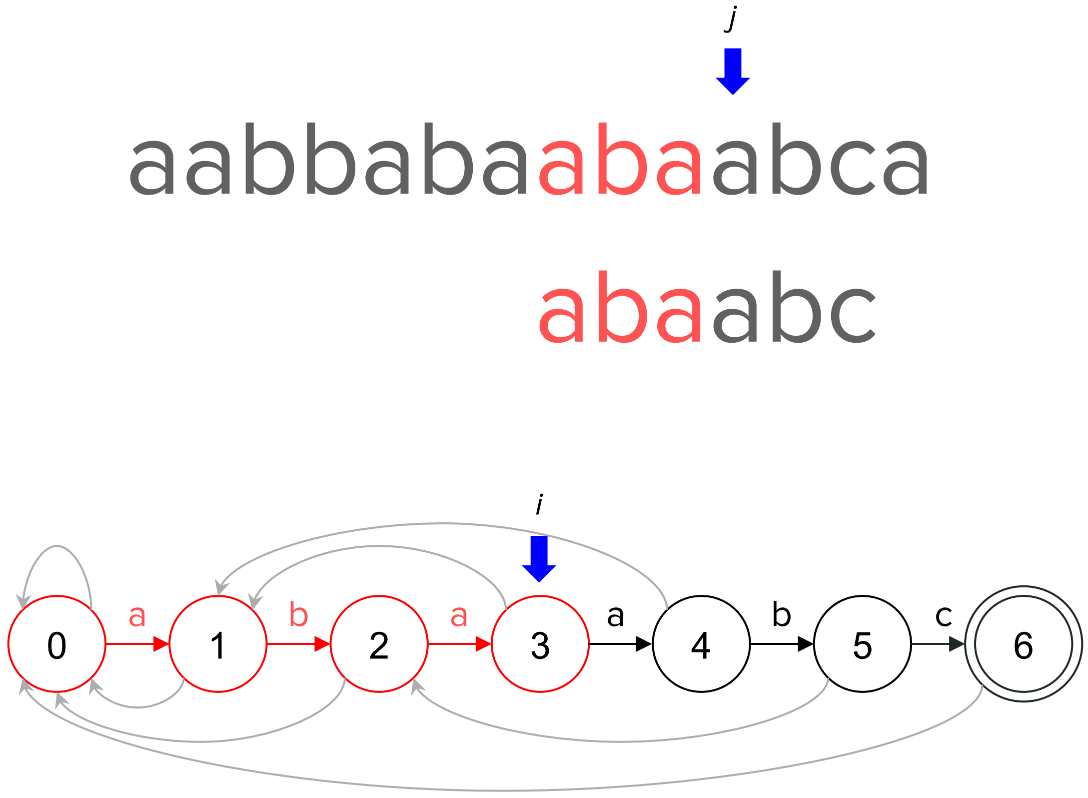

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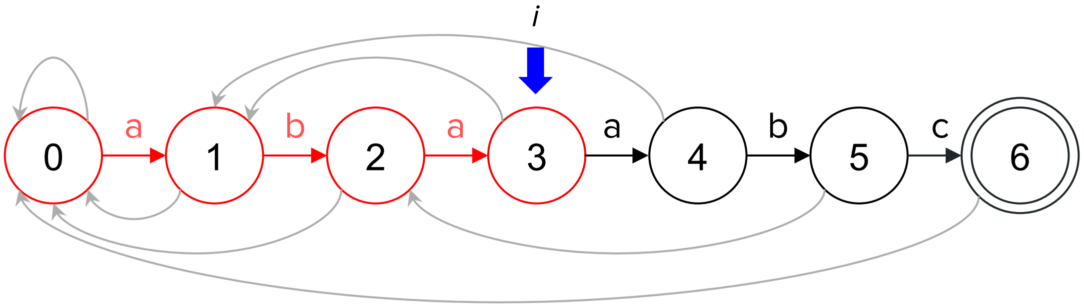

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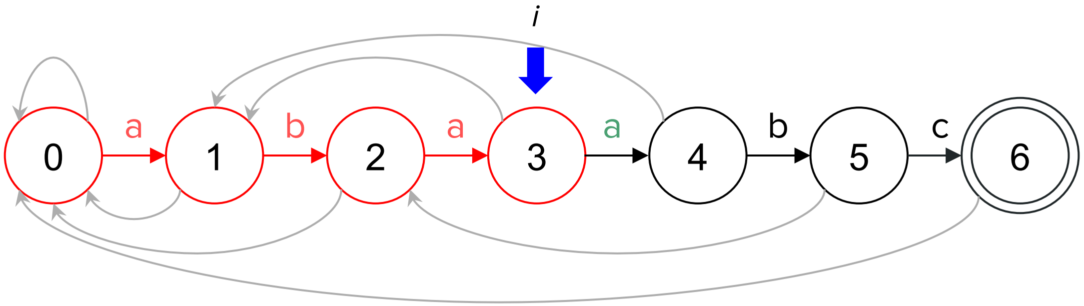

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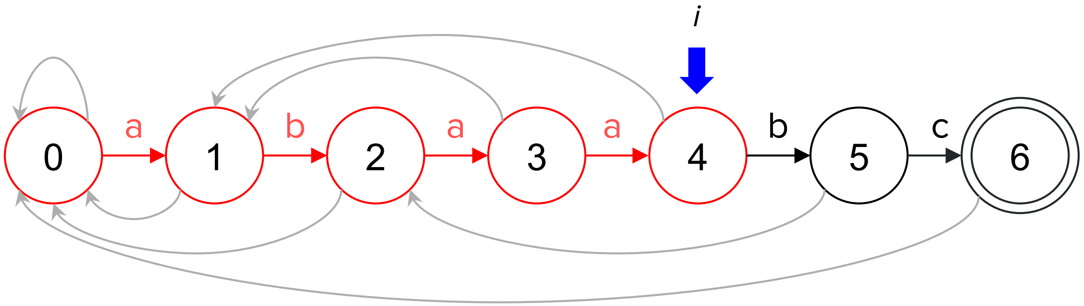
while $j \leq t$ **do**
  if $T_j = P_{i+1}$ **then**
    $j \leftarrow j + 1$
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

j

aabbabaabaabca

abaabc



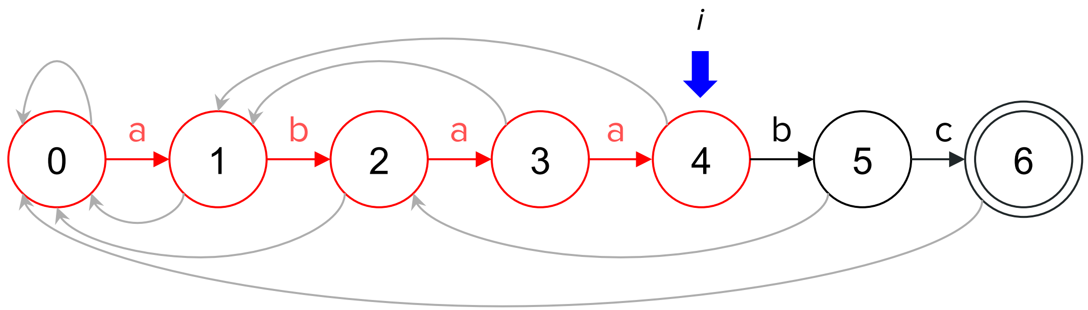
f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
  if $T_j = P_{i+1}$ **then**
    $j \leftarrow j + 1$
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

j

aabbaba**abaa**bca

abaabc



f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \le i \le p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
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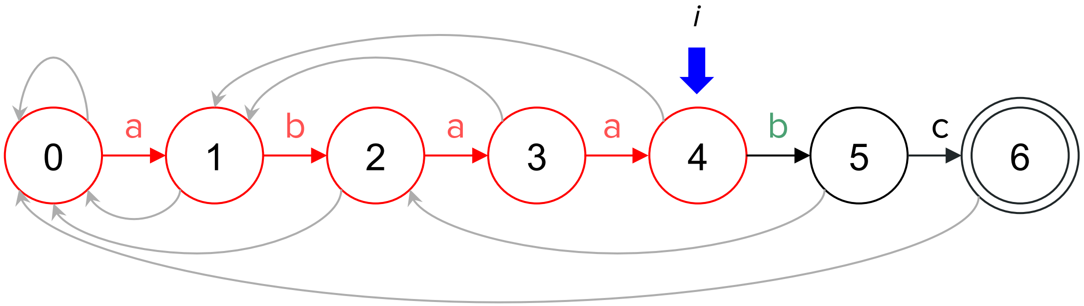
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \ne P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

j

aabbabaabaabca

abaabc



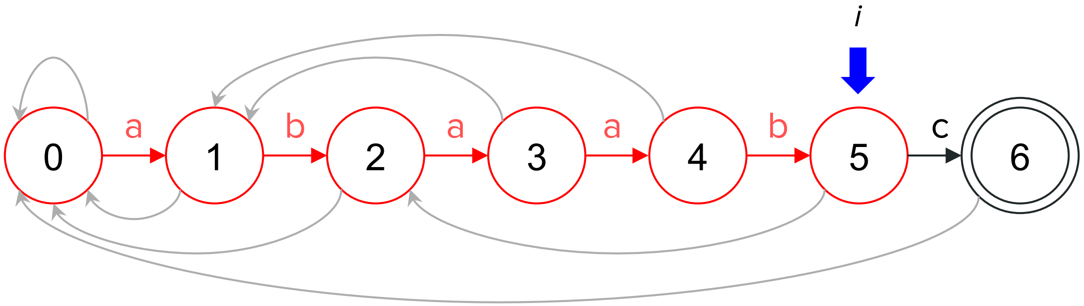
f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
i := current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
    $\boxed{\textbf{if } T_j = P_{i+1}}$ **then**
        $j \leftarrow j + 1$
        M enters state M_{i+1}
        if M is in state M_p **then**
            record $(j - p)$
            M enters state $M_{f(p)}$
        end
    else
        M enters state $M_{f(i)}$
        if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
            $j \leftarrow j + 1$
        end
    end
end

j

aabbabaabaabca

ababc



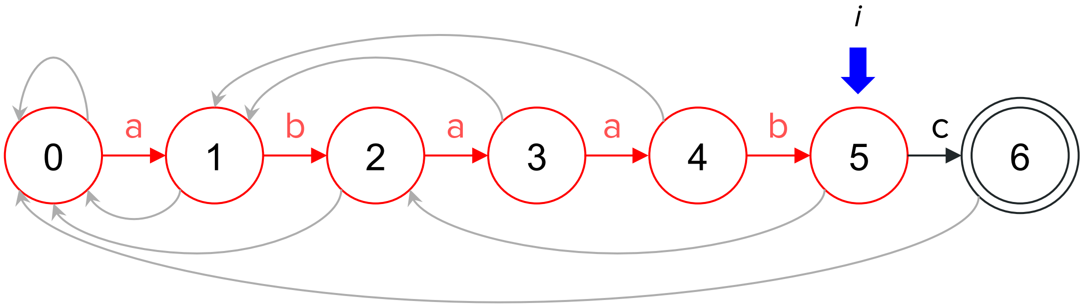
$f(i)$	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
  if $T_j = P_{i+1}$ **then**
    $j \leftarrow j + 1$
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

j

aabbaba**abaabc**a

abaabc



f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
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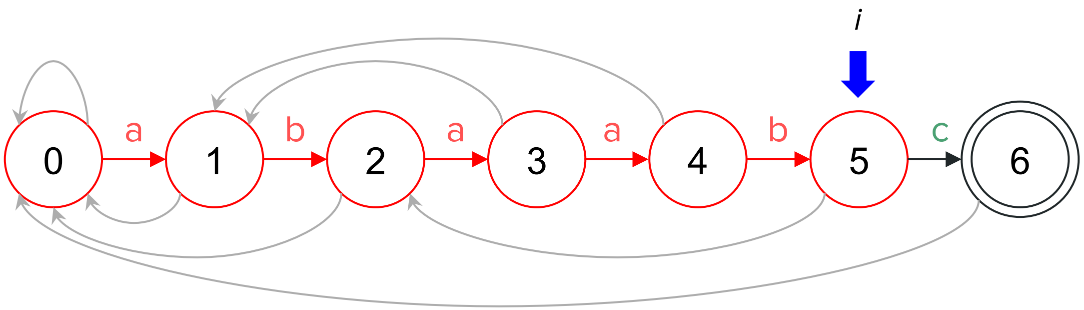
        $j \leftarrow j + 1$
        M enters state M_{i+1}
        if M is in state M_p **then**
            record $(j - p)$
            M enters state $M_{f(p)}$
        end
    else
        M enters state $M_{f(i)}$
        if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
            $j \leftarrow j + 1$
        end
    end
end

j

aabbabaabaabca

abaabc



f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
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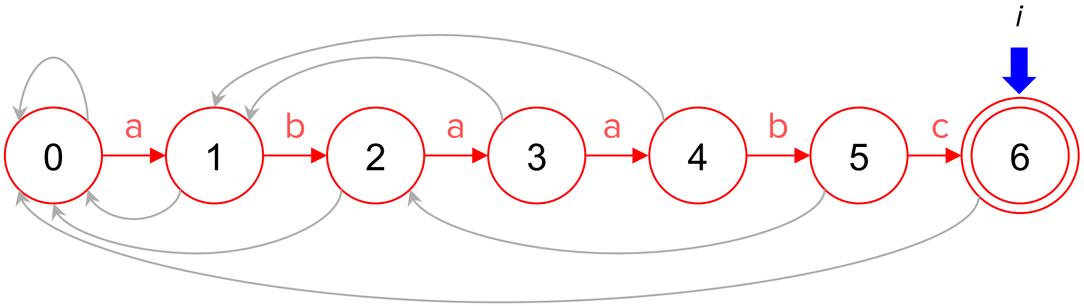
    $j \leftarrow j + 1$
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

j

aabbabaabaabca

abaabc



f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
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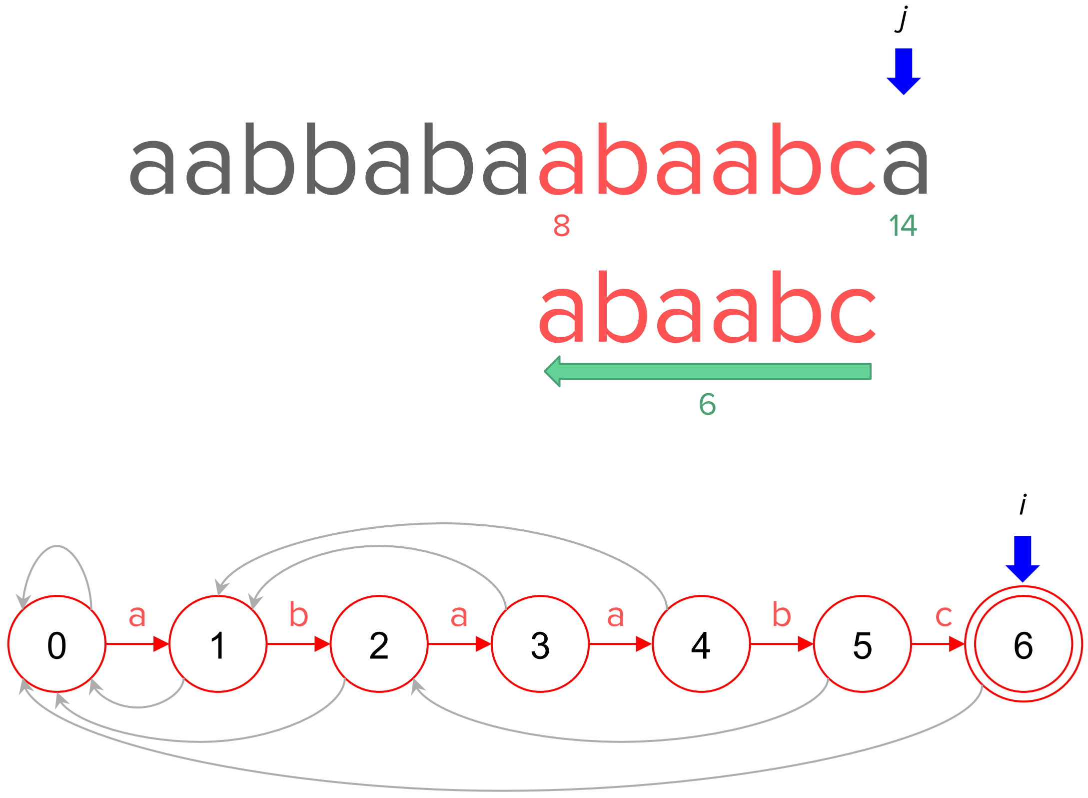
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end



f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \le i \le p$

construct a skeleton DFA M for P using f

M starts in state M_0

$i :=$ current state in M (updated with transitions)

$j \leftarrow 1$

while $j \le t$ **do**

  if $T_j = P_{i+1}$ **then**

    $j \leftarrow j + 1$

    M enters state M_{i+1}

    if M is in state M_p **then**

      record $(j - p)$

      M enters state $M_{f(p)}$

    end

  else

    M enters state $M_{f(i)}$

    if M is in state M_0 **and** $T_j \ne P_{i+1}$ **then**

      $j \leftarrow j + 1$

    end

  end

end

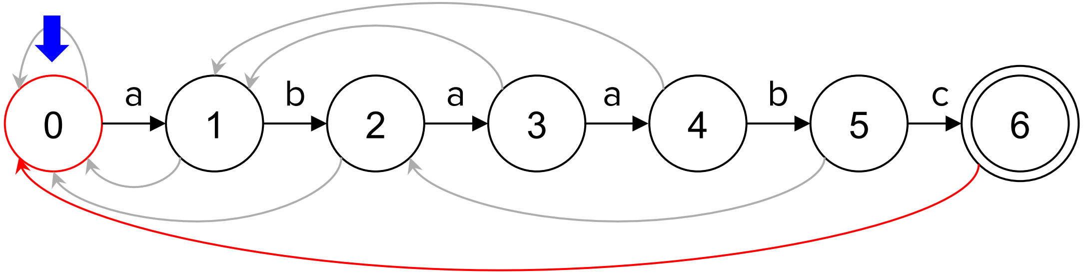
8

j

aabbabaabaabca

abaabc

i



$f(i)$	0	0	1	1	2	0

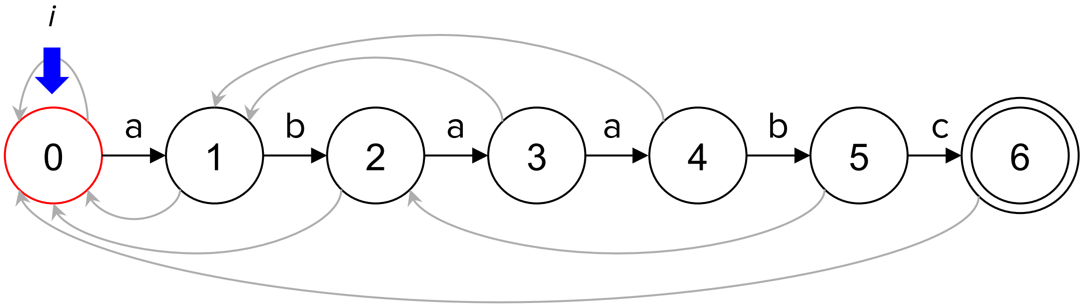
calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
  if $T_j = P_{i+1}$ **then**
    $j \leftarrow j + 1$
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      $\boxed{M \text{ enters state } M_{f(p)}}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

8

j

aabbabaabaabca

ab...



f(i)	0	0	1	1	2	0

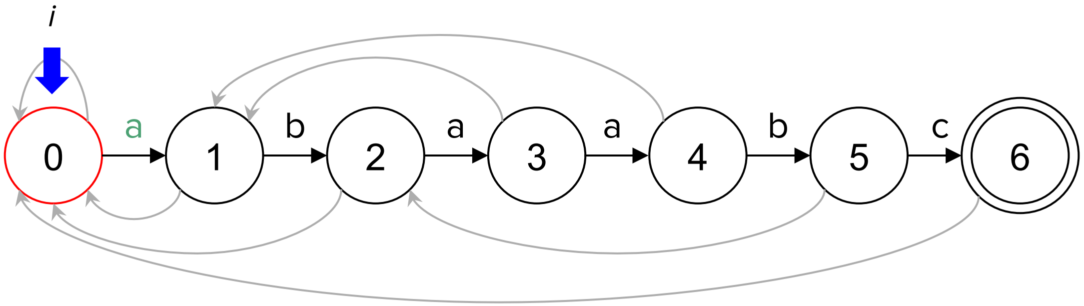
calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
  if $T_j = P_{i+1}$ **then**
    $j \leftarrow j + 1$
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      $\boxed{M \text{ enters state } M_{f(p)}}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

8

j

aabbabaabaabca

ab...



f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
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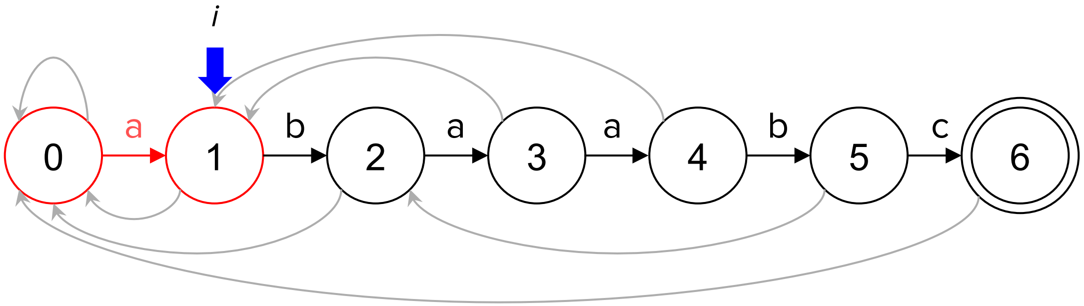
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

j

aabbabaabaabca

ab...



$f(i)$	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \leq t$ **do**
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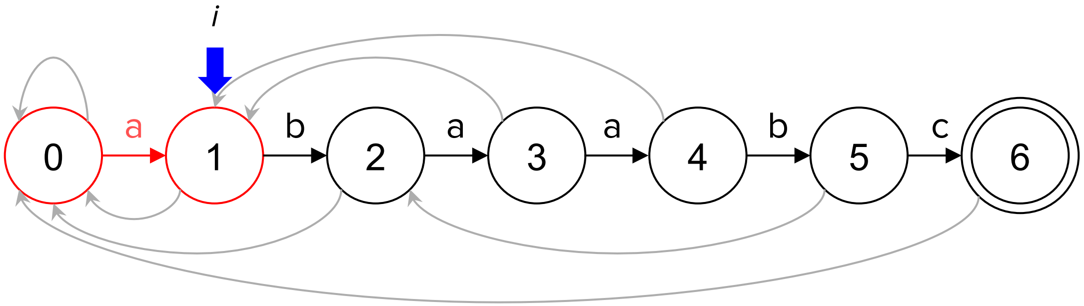
    $j \leftarrow j + 1$
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

j

aabbabaabaabca

ab...



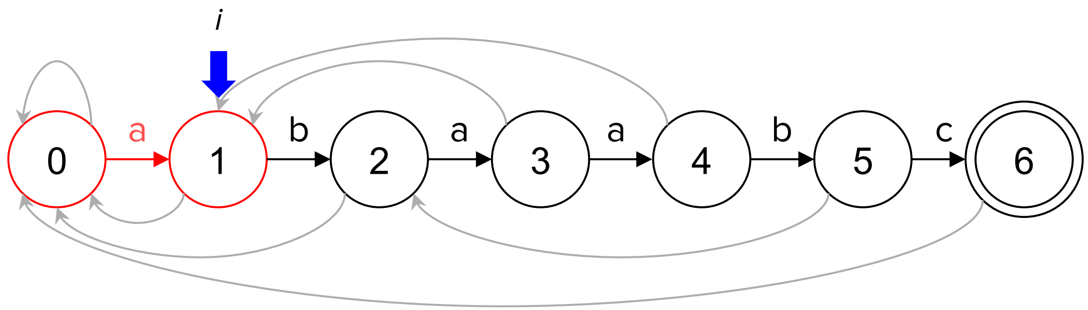
f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \le i \le p$
construct a skeleton DFA M for P using f
M starts in state M_0
$i :=$ current state in M (updated with transitions)
$j \leftarrow 1$
while $j \le t$ **do**
  if $T_j = P_{i+1}$ **then**
    $j \leftarrow j + 1$
    M enters state M_{i+1}
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \ne P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

j

aabbabaabaabca

ab...



$f(i)$	0	0	1	1	2	0

calculate $f(i)$ for $1 \le i \le p$
construct a skeleton DFA M for P using f
M starts in state M_0
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$j \leftarrow 1$
while $j \le t$ **do**
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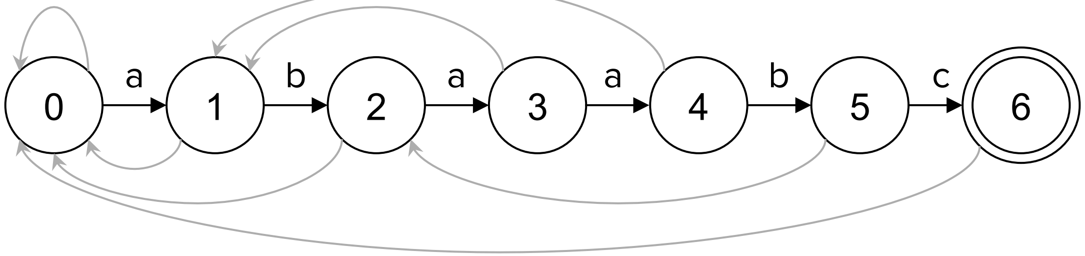
    if M is in state M_p **then**
      record $(j - p)$
      M enters state $M_{f(p)}$
    end
  else
    M enters state $M_{f(i)}$
    if M is in state M_0 **and** $T_j \ne P_{i+1}$ **then**
      $j \leftarrow j + 1$
    end
  end
end

8

aabbabaabaabca

ab...



f(i)	0	0	1	1	2	0

calculate $f(i)$ for $1 \leq i \leq p$
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        if M is in state M_p **then**
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            M enters state $M_{f(p)}$
        end
    else
        M enters state $M_{f(i)}$
        if M is in state M_0 **and** $T_j \neq P_{i+1}$ **then**
            $j \leftarrow j + 1$
        end
    end
end

$\boxed{8}$

Results:

The pattern *P* = “abaabc” occurs once in *T* = “aabbabaabaabca” starting at position **8**.

1	2	3	4	5	6	7	**8**	9	10	11	12	13	14
a	a	b	b	a	b	a	a	b	a	a	b	c	a
							a	b	a	a	b	c	